UT-11-46
IPMU11-0202

A Note on Kahler Potential of Charged Matter in F-theory

Teruhiko Kawano[1], Yoichi Tsuchiya[1] and Taizan Watari[2]

[1] *Department of Physics, University of Tokyo, Tokyo 113-0033, Japan*

[2] *Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa-no-ha 5-1-5, 277-8583, Japan*

Abstract

We study the Kahler potential of charged matter fields, whose profiles have a peak on their matter curve — on an "intersection" of 7-branes, in an F-theory compactification. It is shown that the Kahler potential is exactly given by the integral over the matter curve, but not by the integral over the whole GUT surface of 7-branes.

1 Introduction

Kahler potential of visible sector particles in supersymmetric theories has important consequences in physics. Planck-scale suppressed couplings between the Standard-Model particles and moduli fields or inflaton (may) play the essential role in gravity/anomaly mediated supersymmetry breaking and post-inflationary dynamics of the universe. However, very few claims can be made about such couplings within effective field theories below the Planck scale, and string theory is virtually the only theoretical framework in the market which enables us to make some theoretical progress.

There are already studies along this line [1]. Given the nature of this problem, it is important that string compactification with as many realistic features as possible should be used for such a study. Especially in the context of gravity mediated supersymmetry breaking, it is crucial that we use a framework where flavor violation in Yukawa couplings has already been understood. For this reason, we choose F-theory compactification with charged matter fields of the supersymmetric Standard Model arising from matter curves. After a few years of intense study on flavor structure of Yukawa matrix, it is now known that realistic flavor pattern can be realized in such a compactification, either by tuning one (or a few) complex structure parameter(s) of the matter curve associated with $\mathrm{SU}(5)_{\mathrm{GUT}}$-$\mathbf{10}$ representation fields [2], or by taking appropriate factorization limit of geometry[1] in order to exploit the idea of [9, 10]. See [2, 11] for yet another solution using discrete symmetry.

In this article, we will make a small progress in this program. In F-theory, charged matter fields on "7-brane intersection" are characterized by holomorphic sections of appropriate line bundles on (covering) matter curves [12, 13, 14, 15, 16]. Without a microscopic formulation of F-theory, however, field-theory local models on 7+1 dimensions [17, 18, 19, 20] are the only theoretical tool to calculate low-energy effective Kahler potential (as well as superpotential). Although the relation between the holomorphic sections on matter curves and wavefunctions on GUT divisor is now understood, it would be nice if there are expressions for observables (like F-term and D-term couplings) that are given directly in terms of single-component holomorphic sections on curves, rather than those given indirectly through the (possibly multi-component) wavefunctions on a surface. References [10, 21] exploited holomorphicity in the description of charged matter (and residue integral) and independence of F-term couplings

[1]This is to avoid [3, 2, 4] multiple points of enhanced singularity of E_6 type and D_6 type in the $\mathrm{SU}(5)_{\mathrm{GUT}}$ GUT divisor contributing to the low-energy up-type and down-type Yukawa matrices. It should be noted that factorization of *spectral surface* is not enough because of the problem discussed in [5, 6, 7, 8]. The "factorization" condition needs to be implemented as a constraint on global geometry, to say the least.

on volume moduli, in order to show that the F-term Yukawa couplings are given directly by the holomorphic sections on the matter curves. In this article, we show that the residue integral of [22, 10, 21] can be used also to show that the Kahler potential of charged matter fields on matter curves is also localized on the matter curves.[2]

2 Problem

Local geometry of Calabi–Yau 4-fold X_4 for supersymmetric compactification of F-theory is translated into field theory model on a patch of 7+1-dimensional spacetime [17, 18, 19]. When an elliptic fibration $\pi_X : X_4 \to B_3$ with a section $\sigma : B_3 \to X_4$ has a locus of singularity of type G'' (such as $G'' = A_4$) in the transverse direction at a generic point of a complex surface $S_{\rm GUT} \hookrightarrow B_3 \hookrightarrow X_4$ appearing as an irreducible component of the discriminant of the elliptic fibration, the field theory model for a local geometry in X_4 containing an open local patch $U \subset S_{\rm GUT} \subset X_4$ is a supersymmetric gauge theory on $\mathbb{R}^{3,1} \times U$ with the gauge group G containing G''. The choice of the gauge group G depends on the local geometry, as well as on the level of approximation we want in the field theory model.

In a field theory local model with the gauge group $G = E_6$ that is broken down to $G'' = {\rm SU}(5)_{\rm GUT}$, for example, the symmetry breaking (deformation of singularity) is encoded in a non-trivial configuration of gauge field A_m and Higgs field φ in the $G_{\rm str} = {\rm U}(2)$ subgroup (subalgebra) in E_6 commuting ${\rm SU}(5)_{\rm GUT}$. Chiral multiplets in the $R_I = \mathbf{10}$ and $R_I = \bar{\mathbf{5}}$ representations of $G'' = {\rm SU}(5)_{\rm GUT}$ describing low-energy effective theory below the Kaluza–Klein scale have, in the field theory local model on $\mathbb{R}^{3,1} \times U$, their wavefunction profiles $(\delta A, \delta\varphi)$ in the corresponding irreducible components

$$
\begin{aligned}
{\rm Res}^{G=E_6}_{{\rm U}(2)\times{\rm SU}(5)_{\rm GUT}} \mathfrak{g} &= (\mathbf{2} \otimes \bar{\mathbf{2}}, \mathbf{1}) + (\mathbf{1}, \mathbf{adj.}) + \left[(\mathbf{2}, \mathbf{10}) + (\wedge^2 \mathbf{2}, \bar{\mathbf{5}}) \right] + {\rm h.c.}, \quad (1) \\
&= (\mathbf{2} \otimes \bar{\mathbf{2}}, \mathbf{1}) + (\mathbf{1}, \mathbf{adj.}) + \oplus_I (U_I, R_I). \quad (2)
\end{aligned}
$$

The zero mode wavefunctions $(\delta A^{(0,1)}, \delta\varphi^{(2,0)}) \equiv (\psi, \chi)$ in the irreducible component (U_I, R_I)

[2] Reference [19] wrote down Hermitian inner product of global holomorphic sections of line bundles on matter curves as a "natural" expression, which also follows from simple calculation in the case of single-component matter wavefunction with a Gaussian profile in the direction transverse to the matter curve. Additional evidence in support of the Hermitian inner product on matter curves was obtained [2] at a special slice of matter curves where spectral surface is ramified (7-brane monodromy is non-trivial), and a single-component wavefunction cannot be used under any approximation schemes. This article provides a proof of localization to matter curves along the entire matter curves for any ramified spectral surfaces.

should satisfy

$$\omega \wedge D'\psi + \frac{|\alpha|^2}{2}\rho_{U_I}(\langle\overline{\varphi}\rangle)\chi = 0, \tag{3}$$

$$D''\psi = 0, \tag{4}$$

$$D''\chi - i\rho_{U_I}(\langle\varphi\rangle)\psi = 0. \tag{5}$$

Here, ω is a Kahler form on $U \subset S_{\rm GUT}$, and D' and D'' are the $(1,0)$ and $(0,1)$ part of covariant derivative for fields in the representation U_I of $G_{\rm str}$. See [2] (and [20]) for notations and conventions adopted in this article.

Chiral multiplets in a representation R_I in low-energy effective theory can be characterized only in a geometry along a matter curve $\bar{c}_{(R_I)} \hookrightarrow S_{\rm GUT}$ corresponding to the representation R_I. They form a vector space corresponding to global holomorphic sections of certain line bundles on covering matter curve $\tilde{\bar{c}}_{(R_I)}$ (see [16] for the difference between matter curves $\bar{c}_{(R_I)}$ and their corresponding covering matter curves $\tilde{\bar{c}}_{(R_I)}$; we will not focus on the difference in this article, however.):

$$H^0\left(\tilde{\bar{c}}_{(R_I)}; \mathcal{L}_{G^{(4)}} \otimes K^{1/2}_{\tilde{\bar{c}}_{(R_I)}}\right). \tag{6}$$

Low-energy effective field theory is described by choosing a basis $\{\tilde{f}_{(R_I);i}\}$ of this vector space; the choice of basis of this vector space sets a basis in the chiral multiplets $\{\Phi_{(R_I);i}\}$ of the low-energy physics. $\{\Phi_{(R_I);i}\}$'s (and $\{\tilde{f}_{(R_I);i}\}$'s) have their corresponding wavefunctions (ψ_i, χ_i) for any local patch $U \subset S_{\rm GUT}$ containing a segment of $\bar{c}_{(R_I)}$. The kinetic terms of the chiral multiplets in the representation R_I are given in terms of these wavefunctions as in [2] by

$$K_{\rm eff.} = K^{(R_I)}_{i\bar{j}}\, \Phi^\dagger_{(R_I);\bar{j}} e^{\rho_{R_I}(V)} \Phi_{(R_I);i} + \cdots, \tag{7}$$

$$K^{(R_I)}_{i\bar{j}} \simeq \frac{c_{(U_I,R_I)} M_*^4}{4\pi} \int_U i\omega \wedge \left[\bar{\psi}_{\bar{j}} \wedge \psi_i\right] + \frac{|\alpha|^2}{2}\left[\overline{\chi}_{\bar{j}} \wedge \chi_i\right]. \tag{8}$$

Here, in (8), the zero-mode wavefunctions (ψ_i, χ_i) are treated as ($r_I \equiv$ dim. U_I)-component fields, and $r_I \times r_I$ unit matrix is used for the inner product. The couplings between the visible sector particles such as quarks and leptons and various moduli fields (including Kahler moduli, complex structure moduli, right-handed neutrinos) of the Calabi–Yau 4-fold are encoded in the moduli value dependence of this kinetic mixing matrix[3] $K^{(R_I)}_{i\bar{j}}$.

The problem we face in this article is the following. The expression (8) is given by an integration over a complex *surface* $S_{\rm GUT}$, not precisely on the matter *curve* $\tilde{\bar{c}}_{(R_I)}$, although

[3]Here, the ellipses in the first line are the terms in higher order in the charged chiral multiplets Φ_i. We should note that the equations (3–5) implicitly assume that the zero-mode wavefunctions are infinitesimal fluctuations from the background in $\mathfrak{g}_{\rm str}$ rather than finite deformations. The expression (8) for the kinetic

the wavefunctions (ψ_i, χ_i) become small quickly at points far away from the matter curve. We show by using residue integral (as in [22, 10, 21]) that there is an expression for the kinetic mixing matrix where the integration is carried out exactly on the matter curve.

3 Use of Residue Integral

The zero-mode wavefunctions (ψ, χ) on a complex surface U are related to the sections $\tilde{f}$ on complex curve $\tilde{\bar{c}}_{(R_I)}$ as follows [20, 23, 10, 2]. Hereafter, we drop reference to representations such as U_I, R_I and ρ_{U_I} from zero-mode wavefunctions, sections and background field configuration; the following argument is applied to any one of irreducible components (U_I, R_I), and should be applied separately to these components one by one. Now, first, because the zero-mode wavefunctions (ψ, χ) are vector bundle V_{U_I} (rank $V_{U_I} = \dim U_I = r_I$) valued $(0,1)$-form and $(2,0)$-form, respectively, we can take arbitrary frame for local trivialization of the vector bundle V_{U_I} in describing them. When the unitary frame is replaced by a holomorphic frame,

$$\psi = \mathcal{E}\tilde{\psi}, \qquad \chi = \mathcal{E}\tilde{\chi}, \tag{9}$$

where $\mathcal{E}$ is an $r_I \times r_I$ matrix whose value is in complexification of the structure group $G_{\mathrm{str.}}$, and the zero-mode equations (3–5) become[4]

$$\begin{aligned} \omega \wedge \tilde{D}'\tilde{\psi} + \frac{|\alpha|^2}{2}\tilde{\bar{\varphi}}\,\tilde{\chi} &= 0, & (10)\\ \bar{\partial}\widetilde{\psi} &= 0, & (11)\\ \bar{\partial}\tilde{\chi} - i\widetilde{\varphi}\,\tilde{\psi} &= 0. & (12) \end{aligned}$$

Hermitian inner product of sections of V_I is given in this holomorphic frame by an $r_I \times r_I$ matrix $H = \mathcal{E}^\dagger\mathcal{E}$.

mixing matrix $K^{(R_I)}_{i\bar{j}}$ corresponds to a partial contribution from an open patch $U \subset S_{\mathrm{GUT}}$, and all such contributions from patches along the matter curve $\bar{c}_{(R_I)}$ should be summed up. We should also note that this expression only takes account of the leading order term in the 7+1-dimensional field theory local model Lagrangian in the $1/[M_*^4 \times \mathrm{vol}(S_{\mathrm{GUT}})]$ expansion. It should also be remembered that the low-energy effective Kahler potential should not be obtained by just truncating (forgetting about) various Kaluza–Klein modes, but by integrating them out. This difference should matter for the Kahler potential. In this article, as well as in [2], however, we will take a naive dimensional reduction as a first step to see what is going on.

[4] The background field configuration $\langle\varphi\rangle$ is now simply denoted by φ, as the distinction between zero-mode wavefunctions (ψ, χ) and background configuration in a representation U_I, (A, φ), will already be clear.

The F-term conditions (11, 12) imply that the zero-mode wavefunctions are given by some V_{U_I}-valued field[5] $\tilde{\Lambda}$ and $\tilde{f} \in \Gamma(U; V_{U_I} \otimes K_U)$:

$$\tilde{\psi} = \bar{\partial}\, \tilde{\Lambda}, \qquad \widetilde{\chi} = i\widetilde{\varphi}\, \tilde{\Lambda} + \tilde{f}, \tag{13}$$

with a redundancy of the form $\tilde{\Lambda} \to \tilde{\Lambda} - k$ and $\tilde{f} \to \tilde{f} + i\widetilde{\varphi}\, k$ for arbitrary $k \in \Gamma(U; V_{U_I})$. The remaining D-term condition (10) sets the relation between $\tilde{\Lambda}$ and holomorphic $\tilde{f}$:

$$\omega \wedge H^{-1}\partial \left(H \left(\bar{\partial}\tilde{\Lambda} \right) \right) + \frac{|\alpha|^2}{2} H^{-1} \widetilde{\varphi}^\dagger H \left(i\widetilde{\varphi}\, \tilde{\Lambda} + \tilde{f} \right) = 0. \tag{14}$$

Thus, the zero-mode wavefunctions are determined from $\tilde{f} \in \Gamma(U; V_{U_I} \otimes K_U)$, and the redundancy we mentioned earlier allows us to identify the vector space of zero modes (6) characterized on matter curves with [20, 23, 10, 2]

$$\mathrm{Coker} \left[(\varphi \times) : H^0(\widetilde{C}_{U_I}; \widetilde{\mathcal{N}}_{U_I}) \to H^0(\widetilde{C}_{U_I}; \widetilde{\mathcal{N}}_{U_I} \otimes \tilde{\pi}^*_{C_{U_I}}(K_U)) \right]. \tag{15}$$

$\widetilde{C}_{U_I}$ is the (desingularization of the) spectral surface for the K_U-valued Higgs bundle of the vacuum configuration in the $G_{\rm str}$-U_I representation, and $\widetilde{\mathcal{N}}_{U_I}$ is a line bundle on it, keeping information of 4-form flux in F-theory compactification. Through the projection $\tilde{\pi}_{C_{U_I}} : \widetilde{C}_{U_I} \to U$, the vector bundle V_{U_I} and the line bundle $\widetilde{\mathcal{N}}_{U_I}$ are related as follows: $V_{U_I} = \tilde{\pi}_{C_{U_I}*} \left(\widetilde{\mathcal{N}}_{U_I} \right)$. See [20] for more information.

Having prepared all the necessary material, let us now rewrite the kinetic mixing matrix as follows.[6]

$$\begin{aligned} K^{(R_I)}_{i\bar{j}} \;\simeq\;& \frac{M_*^4}{4\pi} \int_U i\omega \wedge \left[(\partial \tilde{\Lambda}^\dagger_{\bar{j}}) \wedge H(\bar{\partial}\tilde{\Lambda}_i) \right] + \frac{|\alpha|^2}{2} \left[\left\{ \tilde{f}^\dagger_{\bar{j}} - i\tilde{\Lambda}^\dagger_{\bar{j}} (\widetilde{\varphi})^\dagger \right\} \wedge H \left\{ \tilde{f}_i + i\widetilde{\varphi}\tilde{\Lambda}_i \right\} \right] \\ =\;& \frac{M_*^4}{4\pi} \int_U \partial \left(i\omega \wedge \left[\tilde{\Lambda}^\dagger_{\bar{j}} \wedge H(\bar{\partial}\tilde{\Lambda}_i) \right] \right) + \frac{|\alpha|^2}{2} \left[\tilde{f}^\dagger_{\bar{j}} \wedge H \left\{ \tilde{f}_i + i\widetilde{\varphi}\tilde{\Lambda}_i \right\} \right] \\ +\;& \frac{M_*^4}{4\pi} \int_U -i\omega \wedge \left[\tilde{\Lambda}^\dagger_{\bar{j}} \wedge \partial \left(H(\bar{\partial}\tilde{\Lambda}_i) \right) \right] - i\frac{|\alpha|^2}{2} \left[\tilde{\Lambda}^\dagger_{\bar{j}} (\widetilde{\varphi})^\dagger \wedge H \left\{ \tilde{f}_i + i\widetilde{\varphi}\tilde{\Lambda}_i \right\} \right]. \end{aligned} \tag{16}$$

The last line vanishes because of the D-term condition (14), and the first term can be dropped because it is a total derivative. Let us now take a set of local coordinates (u, v) on $U \subset S_{\rm GUT}$, and describe

$$\tilde{f} = \tilde{f}_{uv} du \wedge dv, \qquad \widetilde{\varphi} = \widetilde{\varphi}_{uv} du \wedge dv. \tag{17}$$

[5]This $\tilde{\Lambda}$ wavefunction multiplied by a 4D fluctuation $\phi(x)$ takes its value in the (U_I, R_I) component of $\mathfrak{g}$, and can also be regarded as the generator of an infinitesimal complexified gauge transformation beyond $\mathcal{E}$ in (9).

[6]In order to save space, we will drop the representation-dependent and calculable constant $c_{(U_I,R_I)}$ in (8) in the rest of this article. It is dimensionless, and is of order unity.

Inserting an $r_I \times r_I$ matrix $(\widetilde{\varphi}_{uv})^{\dagger -1}(\widetilde{\varphi}_{uv})^{\dagger}$, we find that

$$
\begin{aligned}
K^{(R_I)}_{i\bar{j}} &\simeq \frac{M_*^4}{4\pi}\int_U \frac{|\alpha|^2}{2}\left[\tilde{f}^{\dagger}_{\bar{j}} \wedge H\left\{\tilde{f}_i + i\widetilde{\varphi}\tilde{\Lambda}_i\right\}\right], \\
&= \frac{M_*^4}{4\pi}\int_U \frac{|\alpha|^2}{2}\left[((\tilde{f}_j)_{uv})^{\dagger}(\widetilde{\varphi}_{uv})^{\dagger -1}(\widetilde{\varphi})^{\dagger} \wedge H\left\{\tilde{f}_i + i\widetilde{\varphi}\tilde{\Lambda}_i\right\}\right], && (18)\\
&= -\frac{M_*^4}{4\pi}\int_U \left[((\tilde{f}_j)_{uv})^{\dagger}(\widetilde{\varphi}_{uv})^{\dagger -1}\omega \wedge \partial\left(H\tilde{\psi}_i\right)\right], && (19)\\
&= \frac{M_*^4}{4\pi}\int_U \left[\partial\left\{((\tilde{f}_j)_{uv})^{\dagger}(\widetilde{\varphi}_{uv})^{\dagger -1}\right\}\omega \wedge H\tilde{\psi}_i\right]; && (20)
\end{aligned}
$$

the D-term condition (14) was used in between (18) and (19) once more.

It is now easy to see that this expression for the kinetic mixing matrix is localized precisely on the matter curve. Because $\tilde{f}$ and $\widetilde{\varphi}$ are holomorphic sections of $K_U \otimes V_{U_I}$ and $K_U \otimes \mathrm{End}V_{U_I}$, respectively, the r_I-component field $[(\widetilde{\varphi}_{uv})^{-1} \cdot \tilde{f}_{uv}]$ remains holomorphic everywhere in $U \subset S_{\mathrm{GUT}}$, except where $\det(\widetilde{\varphi}_{uv})$ vanishes and the inverse matrix is not well-defined, namely, the matter curve for the (U_I, R_I) component. If the matter curve is the simple zero of the determinant, then $\bar{\partial}$ of the r_I-component field $[(\widetilde{\varphi}_{uv})^{-1} \cdot \tilde{f}_{uv}]$ becomes a $(0,1)$-form with its support precisely on the matter curve. This procedure is quite similar to the use of residue integral in [22, 10, 21] for the proof of localization of F-term Yukawa couplings on the matter curve. Although we cannot always expect that holomorphicity plays some role in couplings in the Kahler potential, in this case, we can still use the holomorphicity associated with (15) and residue integral to show the precise localization on the matter curve.

3.1 Residue Integral for Rank-2 Ramified Spectral Surface

To see the power of the expression (20), let us see more in detail how (20) is like when the field theory local model is given by a rank-2 ramified spectral cover C_{U_I}:

$$
(\xi_{uv})^2 + 2c' M_*(M_* u)(\xi_{uv}) - c^2 M_*^2 (M_* v) = 0, \qquad (21)
$$

where (u, v) are local coordinates on $U \subset S_{\mathrm{GUT}}$, and ξ_{uv} is the fiber coordinate of the canonical bundle K_U with the trivialization frame $du \wedge dv$. c and c' are dimensionless constants, which carry the complex structure information of the Calabi–Yau 4-fold of F-theory compactification. In fact, this example is practically important, because the zero-mode wave-functions of chiral multiplets in the $R_I = \mathbf{10}$ representation [$R_I = \bar{\mathbf{5}}$ or $\mathbf{5}$ representation] of $G'' = \mathrm{SU}(5)_{\mathrm{GUT}}$ are determined by the Higgs field background described by (21) near the points of enhanced singularity of type E_6 [resp. type A_6].

Holomorphic sections of *line* bundles $\tilde{f} \in \Gamma(C_{U_I}; \mathcal{N}_{U_I} \otimes \pi^*_{C_{U_I}}(K_U))$ can be translated into the dim $U_I = 2$ component $(2,0)$-forms on $U \subset S_{\rm GUT}$ as follows. First, we take a holomorphic frame e_N for local trivialization of $\mathcal{N}_{U_I}$, so that holomorphic sections $\tilde{f}$ can be written in terms of a holomorphic function $\tilde{f}_{uv}(\xi_{uv}, u)$:

$$\tilde{f} = \tilde{f}(\xi_{uv}, u)\; e_N \otimes \pi^*_{C_{U_I}}(du \wedge dv). \tag{22}$$

A $\mathbb{Z}_2$-Galois transformation $[\xi_{uv} + c' M_*(M_* u)] \to -[\xi_{uv} + c' M_*(M_* u)]$ of the quadratic equation acts on the 2-fold cover (21), and holomorphic functions $\tilde{f}(\xi_{uv}, u)$ on the spectral cover can be written as a sum of $\mathbb{Z}_2$-even part and odd part:

$$\tilde{f}_{uv}(\xi_{uv}, u) = \tilde{f}^{(+)}_{uv}(v,u) + M_*^{-1}(\xi_{uv} + c' M_*(M_* u))\tilde{f}^{(-)}_{uv}(v,u). \tag{23}$$

A 2-component description of the holomorphic sections of $\pi_{C_{U_I}*}(\mathcal{N}_{U_I}) \otimes K_U = V_{U_I} \otimes K_U$ is provided by

$$\tilde{f}_{uv} = \left(\tilde{f}^{(+)}_{uv},\ \tilde{f}^{(-)}_{uv}\right)^T. \tag{24}$$

When the 2-component description of $V_I = \pi_{C_{U_I}*}(\mathcal{N}_{U_I})$ is also given by using the Galois $\mathbb{Z}_2$-even and odd components, the multiplication of $\xi_{uv} = 2\alpha\widetilde{\varphi}_{uv}$ in (15) is represented by [2][7]

$$2\alpha\varphi_{uv} = M_* \begin{pmatrix} -c'(M_* u) & c^2(M_* v) + (c')^2(M_* u)^2 \\ 1 & -c'(M_* u) \end{pmatrix}. \tag{25}$$

Now it is straightforward to calculate

$$(\widetilde{\varphi}_{uv})^{-1}\,\tilde{f}_{uv} = \frac{2\alpha}{M_* c^2(M_* v)} \begin{pmatrix} c'(M_* u) & c^2(M_* v) + (c')^2(M_* u)^2 \\ 1 & c'(M_* u) \end{pmatrix} \begin{pmatrix} \tilde{f}^{(+)}_{uv}(v,u) \\ \tilde{f}^{(-)}_{uv}(v,u) \end{pmatrix}, \tag{26}$$

and hence

$$\bar{\partial}\left\{(\widetilde{\varphi}_{uv})^{-1}\,\tilde{f}_{uv}\right\} = \frac{2\alpha}{M_*^2 c^2}\pi\delta^2(v,\bar{v})d\bar{v} \begin{pmatrix} c'(M_* u) \\ 1 \end{pmatrix} [\tilde{f}_{uv}(\xi_{uv}, u)]|_{\xi_{uv}=0}. \tag{27}$$

This expression is used in (20). This is how we can see that the kinetic mixing matrix is reduced to an expression along the matter curve $v = 0, {}^\forall u$ in $U \subset S_{\rm GUT}$. It is also nice to see that the single component wavefunction on the curve, $[\tilde{f}_{uv}(\xi_{uv}, u)]_{\xi=0}$ to be identified directly with an element of (6, 15), can be pulled out from the 2×2 vector–matrix calculation.

[7]Such a configuration of spectral cover was named "T-brane" in [21].

3.2 Expressions at the E_6 and A_6 points: –use of Painleve equation–

Although it is an important step to show that the expression (8) is reduced to an integral precisely on matter curves, there still remains another important task of rewriting the integrand (20) on the curve in terms of $\tilde{f}_{uv}$ and Kahler and complex structure moduli of the Calabi–Yau 4-fold.

There are not many field-theory local models, however, where the relation between $\tilde{\psi}_i$ and $\tilde{f}_i$ and moduli dependence of the Hermitian inner product H is understood very well. In a field-theory local model where a rank-1 extension of the gauge group provides a good approximation of the 4-fold geometry [17, 18, 19], zero-mode wavefunctions (ψ, χ) have Gaussian profile in the direction transverse to the matter curve, and integration of (8) can be carried out in the transverse direction, first. Such a result—partial contribution to $K^{(R_I)}_{i\bar{j}}$ from segments of matter curves satisfying the condition above—is [2]

$$\Delta K^{(R_I)}_{i\bar{j}} \propto \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\tilde{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}} M_*^4}{2|F|} (2\alpha \tilde{f}_i)_{uv} \right], \tag{28}$$

where we assume that the spectral cover of the field-theory local model is given by $\xi_{uv} - Fv = 0$. This result can be reproduced easily also from (20) by using the relation between $\tilde{\psi}$ and $\tilde{f}_{uv}$ in the Gaussian zero modes.

There is yet another case where we know about the background Higgs bundle configuration and zero-mode wavefunction profiles well [2, 21]. That is the $u = 0$ slice of the field-theory local model given by (21), and will be considered in this subsection. It also describes the configuration of the field-theory local model near the E_6 and A_6 points, the former of which gives rise to the up-type Yukawa couplings. In the rest of this section, we will extend the analysis of [2, 21] by using the residue integral (20) to obtain an easy-to-use expression for the integrand. We assume that the Kahler metric ω is diagonal in the (u, v) coordinates in $U \subset S_{\rm GUT}$, that is, $h_{u\bar{v}} = h_{v\bar{u}} = 0$, and the diagonal terms $h_{u\bar{u}}$ and $h_{v\bar{v}}$ remain constant.

In this special case, at the $u = 0$ slice of the local patch U, a special ansatz can be used for the frame-change 2×2 matrix $\mathcal{E}$ (complexified gauge transformation) in (9):

$$\mathcal{E} = \begin{pmatrix} \underline{\mathcal{E}}^{-1} & \\ & \underline{\mathcal{E}}\, e_{uv} \end{pmatrix}. \tag{29}$$

Here, this ansatz was generalized slightly from [24, 2, 21] by introducing a factor e_{uv} in order to obtain an expression that is covariant under rescaling of local coordinates, $(u', v') =$

$(\lambda_u u, \lambda_v v)$; the factor e_{uv} needs to scale as $e'_{uv} = e_{uv}/(\lambda_u \lambda_v)$. Hitchin equation (D-term BPS condition) turns into

$$\partial_v \bar{\partial}_{\bar{v}} \ln \left(|e_{uv}|^2 |c|^{-2} \mathcal{H}^2 \right) = |\underline{c}|^2 M_*^2 \left[\left(|e_{uv}|^2 |c|^{-2} \mathcal{H}^2 \right) - \frac{|M_* v|^2}{|e_{uv}|^2 |c|^{-2} \mathcal{H}^2} \right] \tag{30}$$

under this ansatz, where $\mathcal{H} \equiv \underline{\mathcal{E}}^\dagger \underline{\mathcal{E}}$ and $\underline{c} \equiv c/\sqrt{h_{u\bar{u}}}$. Requiring that $\mathcal{H}/|v|^{1/2}$ be bounded at large $|v|$ and smooth at $v = 0$ for physical reasons, one can see that this $\mathcal{H}$ (and hence the 2×2 Hermitian inner product) is given by a single function $\mathcal{H}_*$ that depends only on a combination $t' \equiv |M_* v|^{3/2} |\underline{c}|$ [2].

$$|e_{uv}|^2 |c|^{-2} \mathcal{H}^2 = |\underline{c}|^{-2/3} \mathcal{H}_*(t'). \tag{31}$$

Reference [21] pointed out that the differential equation (30) is seen as the Painleve III equation, where $|e_{uv}||c|^{-1}\mathcal{H}/|M_* v|^{1/2}$ is the unknown function and $t = (8/3)t'$ is the variable. From the literature on Painleve equation [25, 26], the numerical value of $H_{0*} \equiv \mathcal{H}_*^2(|v| = 0) \simeq 0.53$ for the bounded solution in [2] turns out to be

$$H_{0*} = 3^{2/3} \left(\frac{\Gamma(2/3)}{\Gamma(1/3)} \right)^2 \simeq 0.531457\ldots. \tag{32}$$

This determines the Hermitian inner product H in (20):

$$H = \begin{pmatrix} \mathcal{H}_*^{-1} \times |\underline{c}|^{1/3}/|c| & \\ & \mathcal{H}_* \times |c|/|\underline{c}|^{1/3} \end{pmatrix} |e_{uv}|. \tag{33}$$

The remaining factor in (20) is the zero-mode wavefunction $\tilde{\psi}$ in the lower ($\mathbb{Z}_2$-odd) component. It is related to $\tilde{f}_{uv}$ in the $u = 0$ slice as [2]

$$\left. \tilde{\psi}^{(-)}_{\bar{v}} d\bar{v} \right|_{v=0} = i \frac{|\underline{c}|^{4/3}}{c^2} \left(-\frac{A_1}{A_0} \right)_* [2\alpha \tilde{f}_{uv}]_{v=0} \, d\bar{v}, \tag{34}$$

where $A_{0,1}$ specifies the linear combination of two independent modes of the zero-mode equations: $\tilde{\chi}^{(+)}(|v|^2) = A_0 + A_1 |M_* v|^2 + \cdots$. The ratio $(A_1/A_0)_* \equiv (A_1/A_0) \times |\underline{c}|^{-4/3}$ should be determined from the requirement that the zero-mode wavefunctions contain only decaying mode (without growing mode) in a region far away from the matter curve. See the appendix C of [2] for more details. Thus, using (27, 33, 34) in (20), we obtain

$$\Delta K^{(R_I)}_{i\bar{j}} \simeq \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\tilde{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}} |e_{uv}| M_*^2}{2|c|^2} (2\alpha(\tilde{f}_i)_{uv}) \right] \sqrt{H_{0*}} \left(-\frac{A_1}{A_0} \right)_*; \tag{35}$$

this result is regarded as a small improvement from the one in the appendix C of [2], in that the factor e_{uv} was introduced, so that this expression is invariant under the rescaling of local coordinates $(u', v') = (\lambda_u u, \lambda_v v)$. Reference [21] pointed out that the damping boundary condition of the zero-mode wavefunctions corresponds to

$$\left(-\frac{A_1}{A_0}\right)_* = \frac{1}{\sqrt{H_{0*}}}. \tag{36}$$

Therefore, an infinitesimal contribution to the kinetic mixing matrix of $\mathrm{SU}(5)_{\mathrm{GUT}}$-**10** representation [of $\mathrm{SU}(5)_{\mathrm{GUT}}$-$\bar{\mathbf{5}}$ and **5** representations] from each E_6 singularity point [A_6 singularity point[8] resp.] becomes quite simple:

$$\Delta K^{(R_I)}_{i\bar{j}} = \omega_{\bar{c}_{(R_I)}} \left[(2\alpha(\tilde{f}_j)_{uv})^* \frac{\sqrt{h^{\bar{u}u}}|e_{uv}|M_*^2}{2|c|^2} (2\alpha(\tilde{f}_i)_{uv}) \right]. \tag{37}$$

4 Discussions

In this article, we have shown that the Kahler potential of charged matter localized on matter curves can be reduced to integrals over the matter curves, but not over the whole GUT surface S_{GUT}. This observation itself is almost obvious in Type IIB Calabi–Yau orientifold compactifications, because there is a zero mode in the world-sheet field $X^M(\sigma, \tau)$ only in the directions where Neumann condition is imposed at both $\sigma = 0$ and $\sigma = \pi$ boundaries. But in F-theory, where such a world-sheet formulation is absent, a proof was necessary for the existence of an expression for the kinetic mixing matrix (like (20)) that is localized along the matter curves.

In order to study how the charged matter Kahler potential (kinetic mixing matrix) depends on Kahler moduli and complex structure moduli, however, further work needs to be done. Charged matter chiral multiplets are characterized by single component holomorphic

[8]Study of [27] shows that the singular elliptic fiber on the "A_6-singularity points" in the small resolution of Calabi–Yau 4-fold is not of I_6 type. We still maintain the same terminology, A_6-singularity point, precisely in the meaning of [20]. After all, without a microscopic formulation of F-theory, there is no persuasive argument (rather than a try-and-error approach) about how to deal with singular geometry *for physics purposes*, and how to derive low-energy physics from singular or desingularized geometry. In order to bypass this complication, physics result of Heterotic string theory was translated into F-theory through duality, and it was discovered that E_6 [A_6 resp.] gauge theory with a smooth Higgs background (without any singularities) reproduces the physics result translated from Heterotic string [16, 20, 2]. This is enough in providing theoretical basis for most of the applications to low-energy physics, if not all. There still remains as a fundamental question, however, what kind of geometry "strings" or membranes see precisely.

sections $\tilde{f}$ along the matter curves, but detailed and analytic description has not been given to the relation between $H\tilde{\psi}$ and $\tilde{f}$, and to how the relation depends on the bulk Calabi–Yau 4-fold moduli. So far, only partial results, (28) and (37), have been obtained.

There is yet another open problem associated with charged matter kinetic mixing matrix (Kahler potential). This problem may be interesting from a theoretical perspective. For charged matter fields arising from the bulk of a stack of multiple 7-branes (so that they are in the adjoint representation of the 7-brane gauge group), chiral multiplets are characterized either as $H^1(S_{\rm GUT}; \mathcal{O})$ or $H^0(S_{\rm GUT}; K_S) = [H^2(S_{\rm GUT}; \mathcal{O})]^\times$. Chiral multiplets of the first kind are described by wavefunctions ψ, and those of the second kind by wavefunctions χ (e.g. [18, 19] in F-theory compactifications). As studied in [28, 29], the adjoint charged matters of the first kind mix with Kahler moduli of the bulk Calabi–Yau, and the adjoints of the second kind with complex structure moduli (and dilaton) of the bulk Calabi–Yau. Once a charged matter chiral multiplets on matter curves (D7–D7 intersection curves in Type IIB orientifold language) come into a picture, however, such a separation may not be maintained; such charged matter fields are described by wavefunctions that do not vanish in either one of ψ and χ. They have character of both kinds. Yet, once we follow the Higgs cascade further down [30] (while ignoring phenomenological applications) so that no non-Abelian symmetry is left unbroken and the F-theory Calabi–Yau 4-fold becomes smooth, the separation between the Kahler moduli Kahler potential and complex structure moduli Kahler potential will emerge, unless the bad approximation of the Kaluza–Klein *truncation* is to be blamed. It is an interesting question how this happens.

Acknowledgements

This work was supported in part by a Grant-in-Aid #23540286 from the MEXT of Japan (TK), by Global COE Program "the Physical Sciences Frontier", MEXT, Japan (YT), by WPI Initiative and by a Grant-in-Aid for Scientific Research on Innovative Areas 2303 from MEXT, Japan (TW).

References

[1] A. Anisimov, M. Dine, M. Graesser, S. D. Thomas, "Brane world SUSY breaking from string / M theory," JHEP **0203**, 036 (2002) [hep-th/0201256]. O. DeWolfe and S. B. Giddings, "Scales and hierarchies in warped compactifications and brane worlds,"

Phys. Rev. D **67**, 066008 (2003) [hep-th/0208123]. S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, "Towards inflation in string theory," JCAP **0310**, 013 (2003) [hep-th/0308055]. M. Grana, T. W. Grimm, H. Jockers and J. Louis, "Soft supersymmetry breaking in Calabi-Yau orientifolds with D-branes and fluxes," Nucl. Phys. B **690**, 21 (2004) [hep-th/0312232]. D. Lust, P. Mayr, R. Richter, S. Stieberger, "Scattering of gauge, matter, and moduli fields from intersecting branes," Nucl. Phys. **B696**, 205-250 (2004) [hep-th/0404134]. Ref. [28]. S. B. Giddings and A. Maharana, "Dynamics of warped compactifications and the shape of the warped landscape," Phys. Rev. D **73**, 126003 (2006) [hep-th/0507158]. D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister and A. Murugan, "On D3-brane Potentials in Compactifications with Fluxes and Wrapped D-branes," JHEP **0611**, 031 (2006) [hep-th/0607050]. S. Kachru, L. McAllister and R. Sundrum, "Sequestering in String Theory," JHEP **0710**, 013 (2007) [hep-th/0703105 [HEP-TH]], and many other works.

[2] H. Hayashi, T. Kawano, Y. Tsuchiya, T. Watari, "Flavor Structure in F-theory Compactifications," JHEP **1008**, 036 (2010) [arXiv:0910.2762 [hep-th]].

[3] J. Marsano, N. Saulina and S. Schafer-Nameki, "F-theory Compactifications for Supersymmetric GUTs," JHEP **0908**, 030 (2009) [arXiv:0904.3932 [hep-th]].

[4] C. Cordova, "Decoupling Gravity in F-Theory," [arXiv:0910.2955 [hep-th]].

[5] R. Tatar, Y. Tsuchiya and T. Watari, "Right-handed Neutrinos in F-theory Compactifications," Nucl. Phys. B **823**, 1 (2009) [arXiv:0905.2289 [hep-th]].

[6] H. Hayashi, T. Kawano, Y. Tsuchiya, T. Watari, "More on Dimension-4 Proton Decay Problem in F-theory – Spectral Surface, Discriminant Locus and Monodromy," Nucl. Phys. **B840**, 304-348 (2010) [arXiv:1004.3870 [hep-th]].

[7] T. W. Grimm, T. Weigand, "On Abelian Gauge Symmetries and Proton Decay in Global F-theory GUTs," Phys. Rev. **D82**, 086009 (2010) [arXiv:1006.0226 [hep-th]].

[8] J. Marsano, N. Saulina and S. Schafer-Nameki, "A Note on G-Fluxes for F-theory Model Building," JHEP **1011**, 088 (2010) [arXiv:1006.0483 [hep-th]].

[9] J. J. Heckman and C. Vafa, "Flavor Hierarchy From F-theory," Nucl. Phys. B **837**, 137 (2010) [arXiv:0811.2417 [hep-th]].

[10] S. Cecotti, M. C. N. Cheng, J. J. Heckman and C. Vafa, "Yukawa Couplings in F-theory and Non-Commutative Geometry," arXiv:0910.0477 [hep-th].

[11] H. Hayashi, T. Kawano and T. Watari, "Constraints on GUT 7-brane Topology in F-theory," arXiv:1112.2032 [hep-th].

[12] G. Curio, "Chiral matter and transitions in heterotic string models," Phys. Lett. B **435**, 39 (1998) [arXiv:hep-th/9803224].

[13] D. E. Diaconescu and G. Ionesei, "Spectral covers, charged matter and bundle cohomology," JHEP **9812**, 001 (1998) [arXiv:hep-th/9811129].

[14] R. Donagi, Y. H. He, B. A. Ovrut and R. Reinbacher, "The particle spectrum of heterotic compactifications," JHEP **0412**, 054 (2004) [arXiv:hep-th/0405014].

[15] R. Blumenhagen, S. Moster, R. Reinbacher and T. Weigand, "Massless spectra of three generation U(N) heterotic string vacua," JHEP **0705**, 041 (2007) [arXiv:hep-th/0612039].

[16] H. Hayashi, R. Tatar, Y. Toda, T. Watari and M. Yamazaki, "New Aspects of Heterotic–F Theory Duality," Nucl. Phys. B **806**, 224 (2009) [arXiv:0805.1057 [hep-th]].

[17] S. Katz and C. Vafa, "Matter From Geometry," Nucl. Phys. B **497**, 146 (1997) [arXiv:hep-th/9606086].

[18] R. Donagi and M. Wijnholt, "Model Building with F-Theory," arXiv:0802.2969 [hep-th].

[19] C. Beasley, J. J. Heckman and C. Vafa, "GUTs and Exceptional Branes in F-theory - I," JHEP **0901**, 058 (2009) [arXiv:0802.3391 [hep-th]].

[20] H. Hayashi, T. Kawano, R. Tatar and T. Watari, "Codimension-3 Singularities and Yukawa Couplings in F-theory," Nucl. Phys. B **823**, 47 (2009) [arXiv:0901.4941 [hep-th]].

[21] S. Cecotti, C. Cordova, J. J. Heckman, C. Vafa, "T-Branes and Monodromy," JHEP **1107**, 030 (2011) [arXiv:1010.5780 [hep-th]].

[22] Martijn Wijnholt, private communication.

[23] R. Donagi and M. Wijnholt, "Higgs Bundles and UV Completion in F-Theory," arXiv:0904.1218 [hep-th].

[24] N. Hitchin, "The Self-duality Equations on a Riemann Surface," Proc. London Math. Soc. (3) **55**, (1987) 59-126.

[25] B. M. McCoy, C. A. Tracy and T. T. Wu, "Painleve Functions of the Third Kind," J. Math. Phys. **18**, 1058 (1977).

[26] A. B. Zamolodchikov, “Painleve III and 2-d polymers,” Nucl. Phys. B **432**, 427 (1994) [hep-th/9409108].

[27] M. Esole and S. -T. Yau, “Small resolutions of SU(5)-models in F-theory,” arXiv:1107.0733 [hep-th].

[28] H. Jockers, J. Louis, “The Effective action of D7-branes in N = 1 Calabi-Yau orientifolds,” Nucl. Phys. **B705**, 167-211 (2005) [hep-th/0409098].

[29] T. W. Grimm, “The N=1 effective action of F-theory compactifications,” Nucl. Phys. **B845**, 48-92 (2011) [arXiv:1008.4133 [hep-th]].

[30] S. Kachru and C. Vafa, “Exact results for N=2 compactifications of heterotic strings,” Nucl. Phys. B **450**, 69 (1995) [hep-th/9505105];
D. R. Morrison and C. Vafa, “Compactifications of F theory on Calabi-Yau threefolds. 2.,” Nucl. Phys. B **476**, 437 (1996) [hep-th/9603161];
M. Bershadsky, K. A. Intriligator, S. Kachru, D. R. Morrison, V. Sadov and C. Vafa, “Geometric singularities and enhanced gauge symmetries,” Nucl. Phys. B **481**, 215 (1996) [arXiv:hep-th/9605200].